Category Theory for AGI Class

Day: Thu, Feb 19 12026

Functor Categories

How to know if red wine makes you live longer?

- Do a pullback on multiple studies
  - What do both papers agree on?

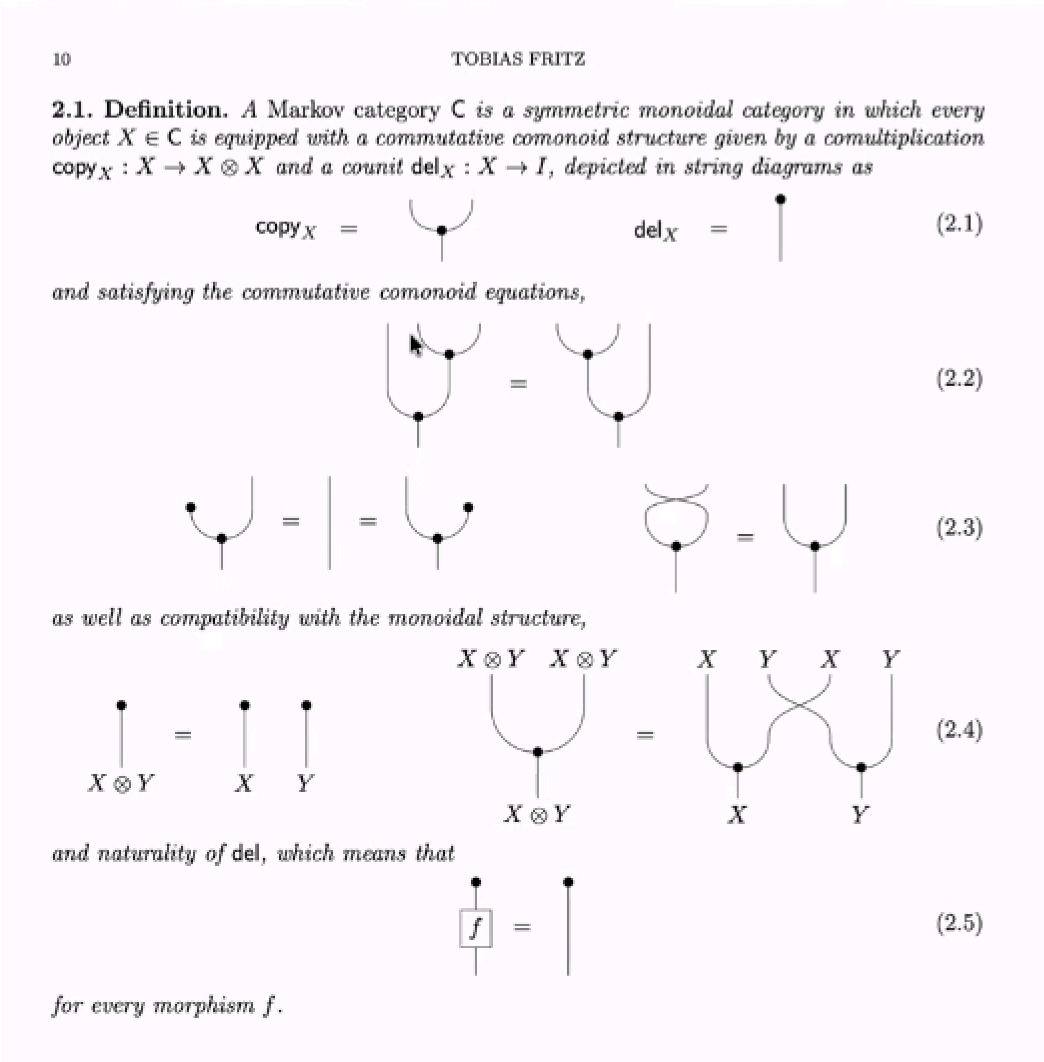
Ex: Soft pullback reconciliation: resveratrol => mitochrondrial biogenesis

Pullbacks:

- an object in a category, such that the domain after two parallel function paths is equivalent

Markov Category

- Symmetric monoidal category where each object can be 'copied' or 'deleted' (pullbacks do not exist)
- If you have an object X, you can get 2 of them. You can also delete X. Can visualized as a string diagram! (symmetric monoidal category)
- Can get all of probability theory just from symmetric monoidal categories?
  - 'A synthetic approach to Markov kernels, conditional independence and theorems on sufficient statistics'



2.1. Definition. *A* Markov category C *is a symmetric monoidal category in which every object* $X \in \mathsf{C}$ *is equipped with a commutative comonoid structure given by a comultiplication* $\mathsf{copy}_X : X \to X \otimes X$ *and a counit* $\mathsf{del}_X : X \to I$*, depicted in string diagrams as*

copy_X = del_X = (2.1)

and satisfying the commutative comonoid equations,

(2.2)

(2.3)

as well as compatibility with the monoidal structure,

(2.4)

and naturality of del*, which means that*

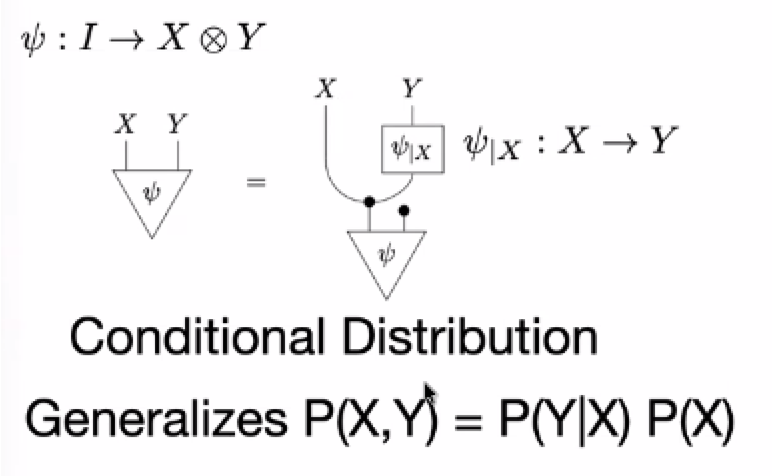
(2.5)

for every morphism f.

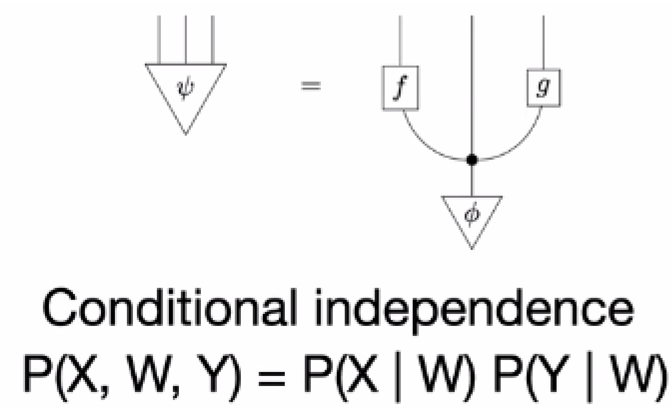
Intuition behind 2.2, 2.3 (read bottom to top)

- 2.2 Doesn't matter which copy you copy
- 2.3 Copy and then deleting one is the same as doing nothing, same with switching copies around (copies are the same)
- 'Think of string diagram as a flow of information through a category'
  - Very suitable to model causal inference
- 2.4 deleting a product is same as deleting them separately, you can duplicate products and it is the same as duplicating things separately
- Deleting a function of something is the same as deleting without the function (function doesn't matter) -> might this be related to information deletion causing entropy creation?

Conditional Distribution generalizes $P(X,Y) = P(Y|X)P(X)$



If you have a joint distribution and delete one of the joints, you can re-create the joint using a conditional slice of the joint.



If you know something about a 3rd variable W, you can factor a joint distribution

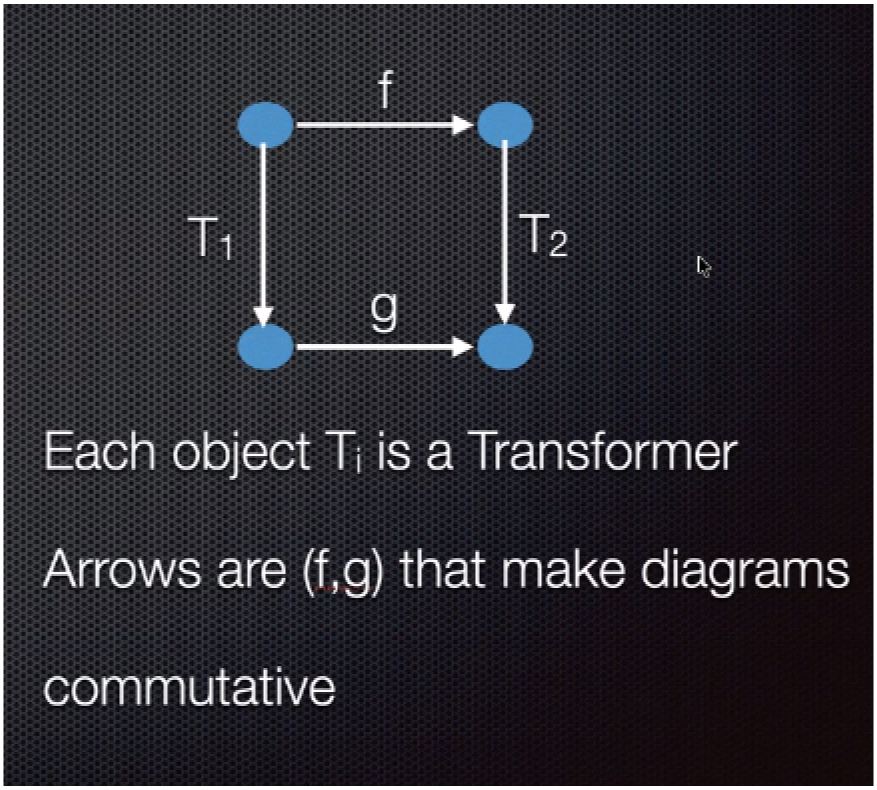
Yoneda Lemma reviewed:

- For any object x in category C, its embedding $C(x, -)$ is a set-valued functor (also called a copresheaf). If each object in these diagrams is a set-valued functor, pullbacks always exist.

Pullbacks in Transformer Category

- We want to prove that in a category of sequence-to-sequence functions, pullbacks exist

- We reduce the problem to pullback in the category of sets.



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Pullbacks abstract over many concrete operations:

- intersection, preimage/inverse image, fibered product (including cartesian product), logical conjunction maximum / meet

Limits over a time series

- You have natural numbers as an indexing category (arrow = +1?)
- Limit must have a projection to each object in the induced diagram,

Diagrammatic Backpropagation

- Extend vanilla backprop to include diagrammatic reasoning, encode compositional constraints
  - "triangular constraints", "rectangular constraints"
  - (basically commutative diagrams)

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Yoneda Lemma generalized to Metric Spaces

- This is core to Diagrammatic Backprop I think, you are trying to impose the yoneda lemma onto the learned structure of transformer

Generalized Metric Space: reflexibity, triangle inequality (not symmetric distances, distances are finite, distances between two different objects cannot be 0)

Metric Yoneda Lemma for generalized metric spaces

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Yoneda takes objects (x) turns them into Functors

- What does the functor do? It takes objects from the category (including x itself) to Sets
- Turns any category into a category of functors, where morphisms are isomorphisms
  - Have all the structure of a set (Topos)

Talks about functors and natural transformations, naturality square.

Talks about Graphs as naturality squares

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Downloaded the textbook and read the first chapter. It covered:

- Categories - Definition
- Transformer as a sequence-to-sequence $\mathbb{R}^{n\times d} \to \mathbb{R}^{n\times d}$
- Commutative Diagrams
- Slice categories as a way to make make "function as object" topoi
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